Tobias Tip: Multifactor Models

Multifactor Pricing Models

Extending our discussion from Week 4, we will now explore multifactor pricing models. The core idea is that we have enough evidence to believe that the CAPM does not hold; that is, there is something else at play in determining expected asset returns beyond just market risk.

The exercise then becomes identifying what these other factors are. These factor models draw on the statistical models of the same name, where the core idea is that we have:

$$\mathbf{X} = \mathbf{LF} + \boldsymbol{\epsilon}$$

Where $\mathbf{X}$ is a $p \times n$ matrix of observations, $\mathbf{L}$ is a $p \times k$ matrix of factor loadings, $\mathbf{F}$ is a $k \times n$ matrix of factors, and $\boldsymbol{\epsilon}$ is a $p \times n$ matrix of idiosyncratic errors.

There are three important things to note about the factor matrix, $\mathbf{F}$:

- The factors are *unobservable* (latent) variables, meaning that we cannot directly measure them, and must instead infer them from the data.
- The dimensionality of the factor matrix is much smaller than that of the observation matrix, i.e., $k \ll p$. This means that we are trying to explain a large number of observations with a small number of factors.
- The factors are orthogonal to both each other and the idiosyncratic errors, i.e., $\mathbf{F}\mathbf{F}^\top = \mathbf{I}$ and $\mathbf{F}\boldsymbol{\epsilon}^\top = \mathbf{0}$. This means that the factors are uncorrelated with each other and with the idiosyncratic errors.

So, to connect this idea to what we learned last week, we can think of the CAPM as a single factor model, where the market return is the only factor. In matrix form, the setup we discussed last week can be represented as:

$$\begin{bmatrix} r_{1,1}r_{1,2} & \cdots & r_{1,n} \\ r_{2,1}r_{2,2} & \cdots & r_{2,n} \\ \vdots & \ddots & \vdots \\ r_{p,1}r_{p,2} & \cdots & r_{p,n} \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{bmatrix} \begin{bmatrix} r_{m,1}r_{m,2} & \cdots & r_{m,n} \end{bmatrix} + \begin{bmatrix} \epsilon_{1,1}\epsilon_{1,2} & \cdots & \epsilon_{1,n} \\ \epsilon_{2,1}\epsilon_{2,2} & \cdots & \epsilon_{2,n} \\ \vdots & \ddots & \vdots \\ \epsilon_{p,1}\epsilon_{p,2} & \cdots & \epsilon_{p,n} \end{bmatrix}$$

Looking at this, we can see that the CAPM is a special case of the multifactor model, and also why not only do we want to test the α of the CAPM individually (for each asset), but also either jointly (for all assets) or via a cross-sectional regression.

With that out of the way, we can now discuss some of the most popular factors in finance. These are the Fama-French factors, and below are the three original factors proposed by Fama and French (FF3):

- **MKT**: Market, already discussed.
- **SMB**: Small Minus Big, which captures the so-called "size effect", whereby small-cap stocks tend to outperform large-cap stocks.
- **HML**: High Minus Low, which captures the "value effect", whereby value stocks (high book-to-market ratio) tend to outperform growth stocks (low book-to-market ratio).

One thing to note about these factors is that they are all "best estimates" of the true (latent) factors. That is, we cannot directly observe the size or value factors, but we can construct proxies for them using observable data.

It's somewhat illustrative to see how these factors are constructed, in particular the methodology behind SMB and HML. A particular emphasis is placed on how we can keep the factors uncorrelated to all other factors.

Let's start with an example: over the last 2–3 years, the technology sector has vastly outperformed the rest of the market. Tech stocks tend to look like growth stocks (low book-to-market), and also tend to be large-cap stocks. Therefore, if we were to construct:

- **SMB** = Small-Cap − Large-Cap Stocks
- **HML** = Value Stocks − Growth Stocks

We would find that not only are these two factors positively correlated with each other (because tech stocks are both large-cap and growth), but also correlated with the market factor (because tech stocks have done well recently). This violates the orthogonality condition that we discussed earlier.

To get around this, Fama and French do what's called a 2×3 sort. The idea is that *before* we construct the factors, we first sort all stocks into 2 different size groups (Small and Big), and then within each

size group we sort stocks into 3 different book-to-market groups (Low, Medium, High). This gives us a total of 6 portfolios:

- Small-Low (SL)
- Small-Medium (SM)
- Small-High (SH)
- Big-Low (BL)
- Big-Medium (BM)
- Big-High (BH)

With these 6 portfolios, we can now construct the SMB and HML factors as follows:

- **SMB** $= \frac{1}{3}(\text{SL} + \text{SM} + \text{SH}) - \frac{1}{3}(\text{BL} + \text{BM} + \text{BH})$
- **HML** $= \frac{1}{2}(\text{SH} + \text{BH}) - \frac{1}{2}(\text{SL} + \text{BL})$

This construction ensures that both SMB and HML are uncorrelated with each other and with the market factor.

Testing Multifactor Models

This section is intentionally brief, since the testing procedure is very similar to what we discussed last week for the CAPM. The main difference is that instead of a single factor (market), we now have multiple factors. Therefore, the time-series regression for each asset i becomes:

$$r_i = \alpha_i + \beta_{\text{MKT}} r_{\text{MKT}} + \beta_{\text{SMB}} r_{\text{SMB}} + \beta_{\text{HML}} r_{\text{HML}} + \epsilon$$

$$\mathbb{E}[r_i] = \alpha + \beta_{\text{MKT}} \mathbb{E}[r_{\text{MKT}}] + \beta_{\text{SMB}} \mathbb{E}[r_{\text{SMB}}] + \beta_{\text{HML}} \mathbb{E}[r_{\text{HML}}]$$

And therefore we should test whether $\alpha_i = 0$ for each asset i. Similarly, for the cross-sectional regression, we have:

$$\mathbb{E}[r_i] = \gamma_0 + \gamma_{\text{MKT}} \beta_{\text{MKT, i}} + \gamma_{\text{SMB}} \beta_{\text{SMB, i}} + \gamma_{\text{HML}} \beta_{\text{HML, i}} + \epsilon$$

And we should test whether $R^2 = 1$, and we also are able to recover the implied factor premia γ_{MKT}, γ_{SMB}, and γ_{HML}. What is a premium?

You'll notice from the notes that we often refer to the expected returns of the factors as "premia" or "risk premia". The idea here is that these factors are a source of systematic risk, i.e., you could be taking no market risk (MKT), but still be exposed to size risk (SMB) or value risk (HML).

The idea is that these factors are a predictable source of risk – a common factor impacting all stocks – and therefore investors should be compensated for bearing this risk. This compensation is what we refer to as the "risk premia".

Money Stuff on the Illiquidity Risk Premia

Bloomberg Opinion

Private credit

Hedge funds are the new investment banks.

One theme around here is that the big multistrategy hedge funds — Citadel, Millennium, Point72, Balyasny, etc. — are slowly becoming what the big investment banks were 20 years ago. Before 2008, firms like Goldman Sachs Group (1) helped customers trade stocks and bonds, (2) invested money for clients in stocks and bonds, and (3) did a lot of complicated trades that sort of sat in between flow trading and investing. Index funds had to buy and sell big piles of stocks whenever the index rebalanced, and the investment banks would help them do that. Shareholders of merger targets wanted to cash in before their deals closed, and Goldman's famous risk arbitrage desk would help them do that. There were basis trades and deal-contingent hedges. Sometimes risky companies needed to borrow money in structured ways to do complicated things, and Goldman's also-famous special situations group would lend them the money. All sorts of complicated bespoke services that Goldman could provide to the market, not because it got paid a fee but because those services carried an expected return as compensation for their risk, and Goldman was happy to take that risk to earn that return.

And then 2008 happened, all the big investment banks either became or were acquired by regulated banks, and that whole genre of trade largely withered away. This weekend, the Wall Street Journal's Miriam Gottfried profiled Alan Waxman, the chief executive officer of private credit firm Sixth Street, who previously ran the special situations group at Goldman:

> At the height of the financial crisis in 2008, Goldman became a bank holding company, making it eligible for emergency government loans but subjecting it to stricter regulation.
>
> The next day Waxman decided to leave. He and his partners launched what eventually became Sixth Street, with the goal of replicating the flexibility of investing from Goldman's balance sheet.

That sort of trade worked at Goldman, until one day in 2008 it didn't, so it immediately started cropping up elsewhere. The basis trades and index-rebalancing trades and risk-arbitrage trades and even deal-contingent hedges are now often done by hedge funds, and those hedge funds — which are lightly regulated, highly leveraged and financed in the capital markets, not with deposits — look a bit like the independent investment banks did 20 years ago.

If you're in the general-purpose business of finding new risk premia to collect in financial markets, at some point you will have the same realization that Goldman had with SSG, which is "people want to borrow money, we have money, rigid old-school banks won't lend to them, but we can, and we'll get paid a lot." Gottfried writes:

> Waxman started as a junior analyst at Goldman in 1998. He worked in the special situations unit, which invested the firm's own money in areas such as corporate equity and debt, real estate and infrastructure.
>
> Early on, he was sent to review a portfolio of loans Goldman wanted to buy. The loans, to radio and television companies, were made by a nonbank lender. Waxman noticed the lender could charge

> hefty interest rates making loans banks wouldn't because it was willing to account for the value of the borrowers' underlying assets, not just its cash flows.
>
> "By looking at things differently than a strict bank, we were able to get comfortable with making loans to these businesses," Waxman says. "I thought: Why can't this be applied across other industries?"

Nowadays that is called "private credit." It is much more industrialized than it was in the 1990s, but even now, if you run a big multistrategy hedge fund, you will occasionally have similar realizations. And so the Financial Times reports:

> Big hedge funds are pushing into private credit as they seek to establish themselves as diversified financial institutions, with Millennium Management, Point72 and Third Point all looking to launch new funds and strategies. ...
>
> "Hedge funds are in the asset gathering business," said one leading banker to hedge funds. "The boom in private credit has really attracted their attention."

Yes, it is hard to run a general-purpose financial services firm and not have a pot of money to lend to companies when the price is right.